

**Semester – II**  
**Paper – I (B)- Advanced Abstract Algebra -II**

Unit- I

Normal series, solvable groups, nilpotent groups and related results.(15 lectures)

Unit – II

Structure theorems of groups, direct products, finitely generated abelian groups, Invariants of a finite abelian group. (15 lectures)

Unit - III

Modules, definition and examples, sub modules and direct sums, R – homomorphisms, quotient modules, completely reducible modules, free modules, representation and rank of a linear mapping. (15 lectures)

Unit – IV

Noetherian and Artinian modules and rings, Wedderburn – Artin theorem, uniform modules, primary modules, Noether –Lasker theorem similarity of linear transformations (15 lectures)

Unit – V

Reduction to triangular form, nilpotent transformation, index of nil potency, invariants of a nilpotent linear transformation, Jordan blocks, Jordan forms, (i) smith normal form (15 lectures)

**Text Book:**

P. B. Bhattacharya, S. K. Jain and S. R. NagPaul, Basic Abstract Algebra, Cambridge University Press, Indian Edition, 1997  
 Chapter 15, 16, 17 and 18 complete

**Reference Books:**

1. I. N. Herstein: Topics in algebra, Wiley Eastern Ltd., New Delhi, 1975.
2. S. Lang: Algebra, 3<sup>rd</sup> edition, Addison-Wesley, 1993.
3. I. S. Luther and I.B.S. Passi: Algebra, Vol. I and Vol. II Narosa, New Delhi.
4. D. S. Malik, J. N. Mordeson and M. K. Sen: Fundamentals of Abstract Algebra, Mc Graw-Hill, and International Edition, 1997.
5. S. K. Jain, A. Gunawardena and P. B. Bhattacharya: Basic Linear Algebra with MATLAB, Key College Publishing (Springer-Verlag), 2001.
6. J. B. Fraleigh, a first course in Abstract Algebra, Narosa Publications.

**Semester – II**  
**Paper – II (B) Real Analysis -II**

Unit – I

Measure on the real line. Lebeque outer measure, measurable sets. Regularity. Measurable functions. Borel and lebeque measurability. Examples. (15 lectures)

Unit – II

Integration of functions of a Real variable. Integration of a simple function. Integration of non-negative functions. The general integral. Integration of series. Examples. (15 lectures)

Unit – III

Riemann and Lebeque Integrals, Differentiation. The four derivatives, Functions of bounded variations. Lebeque's differentiation theorem differentiation and Integration. Examples. (15 lecturer)

Unit – IV

Abstract Measure spaces. Measures and outer measures Extension of a measure. Uniqueness of the extension. Completion of a measure spaces. Integration with respect to a measure. Examples. (15 lecturer)

Unit – V

The  $L^p$  spaces. Convex functions. Jensen's inequality. The inequalities of Holder and Minkowski Completeness of  $L^p(\mu)$  Convergence in measure. Almost uniform convergence. Examples. (15 lecturer)

**Text Book:**

G. de Barra, Measure Theory and Integration. Wiley Eastern Ltd. 1981. Reprint 2003.

Articles: 2.1-2.5, 3.1 – 3.4, 4.1, 4.3 - 4.5, 5.1 – 5.6, 6.1 – 6.5, 7.1 and 7.2

**Reference Books:**

1. P. K. Jain and P. V. Gupta, Lebesgue Measure and Integration, New Age International (P) Ltd. Publication New Delhi. 1986 (Reprint 2000)
2. P. R. Halmos, Measure Theory, Von No strand, Princeton 1950
3. R. G. Bartle, The elements of Integration, John Wiley, New York 1966.
4. I. K Rana, An Introduction to measure and Integration, Narosa, Delhi 1997.

**Semester – II**  
**Paper – III (B) Topology -II**

Unit – I

Separation axioms,  $T_0, T_1, T_2$ , space their properties and characterizations, regular spaces,  $T_3$  spaces, characterizations, Hereditariness of these concepts, completely regular and tychonoff spaces and their characterizations. (15 lectures)

Unit – II

Normal spaces and  $T_4$  spaces, Urysohn's lemma, Tietze theorem on normal spaces (without proof) cover, point finite cover, shrinkable cover of topological spaces and their properties, accountability properties, second countable spaces, Lindelöf spaces and their properties. (15 lectures)

Unit – III

Compactness. Definition and examples, characterization of compactness, sequentially and countably compact spaces, locally compact spaces and their properties, compactification, one point compactification, Stone-Čech compactification. (15 lectures)

Unit – IV

Paracompactness, local finiteness, Metrizable spaces, Lebesgue covering lemma, Urysohn's metrization theorem, metrizeability of  $T_0$  spaces, (15 lectures)

Unit – V

Connected spaces, mutually separated sets, characterizations and properties of connected spaces, components, simple chain, path wise and local connectedness. (15 lectures)

**Text Book:** Stephan Willard: General Topology Addison Wesley publication Co. (1970)

Chapter 5 (Complete) chapter- 6 (Section 17.1 to 19.5, 20.1 to 20.10)

Chapter 7 (Section 22.1 to 23.2, 23.5), Chapter 8 (Section 26.1 to 27.13)

**Reference Books:**

1. Steen & J. Seebach: Counter examples in Topology, Holt, Rinehart and Winston, N, Y. (1970).
2. W. J. Pervin: Foundation of general Topology Academic press N.Y.
3. S. T. Hu. : Elements of general Topology, Holden.
4. James Munkres: Topology, A first course, Prentice Hall of India Pvt. Ltd.

**Semester - II**  
**Paper – IV (B) Complex Analysis -II**

Unit – I

Compactness and convergence in the space of Analytic functions:  
 Spaces of analytic functions; The weierstrass factorization theorem; factorization  
 of the sine function; The gamma function; The Riemann zeta function.  
 (15 lectures)

Unit – II

Harmonic functions:  
 Basic properties of Harmonic functions and comparison with analytic function;  
 Harmonic functions on a disk; Poisson integral formula; positive harmonic  
 functions. (15 lectures)

Unit – III

Entire functions; Jensen's formula; The Poisson-Jenson formula; The genus and  
 order of an entire function. Hadamard factorization Theorem; (15 lectures)

Unit – IV

Univalent functions; the class S; the class T; Bieberbach conjecture; sub class  
 of s; (15 lectures)

Unit – V

Analytic continuation: Basic concepts; special functions. (15 lectures)

**Text Books:**

1. John B. Conway; Functions of one complex variable, Narosa Publishing House, 1980.
2. Herb Silverman; Complex Variables Houghton Mifflin Company Boston 1975.

Unit – I : Chapter – VI: § 2,5,6,7 & 8 in [1]

Unit – II : Chapter – X: § 1& 2 in [1]  
 And Chapter- X: § 10.1, 10.2 & 10.3 in [2]

Unit – III : Chapter- XI: § 1,2 & 3 in [1]

Unit – IV : Chapter XII: § 12.1& 12.2 in [2]

Unit – V : Chapter – XIV: § 14.1 & 14.2 in [2]

**Reference Books:**

1. L. V. Ahlfors: Complex Analysis, McGraw-Hill International Editions, 1979.
2. Ruel V. Churchill and J. W. Wran; Complex variables and applications, McGraw- Hill publishing Company – 1990.

**Semester – II****Paper – V-a (B) Differential Equations - II**

## Unit- I

Preliminaries, Basic Facts: Superposition principles, Lagrange Identity, Green's formula, variation of constants, Liouville substitution, Riccati equations Prufer Transformation. Higher order linear equations. (15 lectures)

## Unit – II

Maximum Principles and their extensions, Generalized maximum principles, initial value problems, boundary value problems. (15lectures)

## Unit –III

Theorems of Sturm; Sturm's first comparison theorem, Sturm's separation theorem, Sturm's second comparison theorem. (15lectures)

## Unit – IV

Sturm-Liouville boundary Value Problems: definition, eigenvalues, eigenfunctions, orthogonality. (15lectures)

## Unit – V

Number of zeros, Non oscillatory equations and principal solutions, Non oscillation theorems. (15 lectures)

**Text Books:**

1. Philip Hartman: Ordinary differential Equations, 2<sup>nd</sup> Edition SIAM, 2002. Chapter – XI: Article 1 to 7. Chapter – 4 – article 8 only.
2. M. H. Protter and H. F. Weinberger, Springer: Maximum Principles in Differential Equations – Springer Verlag, New York, Inc, 1984. Chapter 1. Articles 1 to 4.

**Reference Books:**

1. W. T. Reid: ordinary differential Equations, John Wiley N.Y. (1971).
2. E. A. Coddington and N. Levinson: Theory of Ordinary differential Equation, McGraw-Hill, New York, (1955).

## Semester – II

### PAPER V–b (B): ADVANCED DISCRETE MATHEMATICS -II

#### Unit – I

Definiton of (undirected) graph, paths, circuits, cycles and subgraphs, degree of a vertex connectivity, planar graphs and their properties.

#### Unit-II

Trees, rulers formula for connected planar graphs. Complete graphs, Kuratowski's theorem (statement only) spanning trees, cutsets, fundamental cut-sets and cycles, minimal spanning trees and Kruskal's (statement only) algorithm, matrix representation of graphs,

#### Unit-III

Euler's theorem on the existence of Eulerian paths and circuits, directed graphs, in degree and out degree of a vertex, weighted undirected graphs, strong connectivity, directed trees, search trees,

#### Unit-Iv

Introductory computability theory:

Finite state machines and their transition table diagrams, equivalence of finite state machies, reduced machines, homomorphism, finite automata, acceptors, no-deterministic finite automata.

#### Unit-V

Grammers and languages: Phase structure grammars, rewriting rules, derivations, sentential forms, language generated by a grammar, regular, contest free and contest sensitive grammers and languages.

#### Reference Books:

1. **J. P. Tremblay and R. Manohar:** Discrete Mathematical structures with Applications to Computer science, McGraw-Hill Book Co., 1997.
2. **Seymour Lipschutz:** Finite Mathematics, McGraw-Hill, New York.
3. **S. Wiitala:** Discrete Mathematics - A Unified Approach, McGraw-Hill.
4. **J. E. Hhocroft and J.D. Ullman:** Introduction to Automata Theory, Languages and Computation, Narosa, New Delhi.
5. **C. L. Liu:** Elements of discrete Mathematics, McGraw-Hill Book Co.

**Semester – II****Paper – V - c (B)- Differential Geometry of Manifolds - II**

Riemannian manifolds. Riemannian connection. Curvature tensors. Sectional Curvature. Schur's theorem. Geodesics in a Riemannian manifold. Projective curvature tensor. Conformal curvature tensor.

Submanifolds & Hypersurfaces. Normals. Gauss' formulae. Weingarten equations. Lines of curvature. Generalized Gauss and Mainardi-Chodazzi equations.

Almost Complex manifolds. Nenhuis tensor. Contravariant and covariant almost analytic vector fields. F-connection.

**Reference Books:**

1. R.S. Mishra, A course in tensors with applications to Riemannian Geometry, Pothishala (pvt.) Ltd., 1965.
2. R.S. Mishra, Structures on a differentiable manifold and their applications, Chandrama Prakashan, Allahabad, 1984.
3. B. B. Sinha, An Introduction to Modern Differential Geometry, Kalyani Publishers, New Delhi, 1982.
4. K. Yano and M. Kon, Structure of manifolds. World Scientific, 1984.